Charging Scheduling of Electric Vehicles with Local Renewable Energy under Uncertain Electric Vehicle Arrival and Grid Power Price

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Abstract—In the paper, we consider delay-optimal charging scheduling of the electric vehicles (EVs) at a charging station with multiple charge points. The charging station is equipped with renewable energy generation devices and can also buy energy from power grid. The uncertainty of the EV arrival, the intermittence of the renewable energy, and the variation of the grid power price are taken into account and described as independent Markov processes. Meanwhile, the charging energy for each EV is random. The goal is to minimize the mean waiting time of EVs under the long term constraint on the cost. We propose queue mapping to convert the EV queue to the charge demand queue and prove the equivalence between the minimization of the two queues' average length. Then we focus on the minimization for the average length of the charge demand queue under long term cost constraint. We propose a framework of Markov decision process (MDP) to investigate this scheduling problem. The system state includes the charge demand queue length, the charge demand arrival, the energy level in the storage battery of the renewable energy, the renewable energy arrival, and the grid power price. Additionally the number of charging demands and the allocated energy from the storage battery compose the two-dimensional policy. We derive two necessary conditions of the optimal policy. Moreover, we discuss the reduction of the two-dimensional policy to be the number of charging demands only. We give the sets of system states for which charging no demand and charging as many demands as possible are optimal, respectively. Finally we investigate the proposed radical policy and conservative policy numerically.

Index Terms—Electric vehicle, charging scheduling, renewable energy, Markov decision process.

I. INTRODUCTION

As an important method of operation to mitigate the shortage of the fossil fuel and severe environmental problems, the electric vehicle (EV) technology has attracted much interest in recent years. Compared to conventional vehicles, EVs have advantages in the following aspects: energy efficiency, eco-effect, performance benefits, and energy independence [1]. However, a fuel driven vehicle can produce less CO2 than an EV if the charging energy is entirely produced by coal-fired power plants [2]. Thus, the renewable energy (e.g., solar or wind energy [3]) should be the energy source of the EVs fully or at least partially to achieve the real environmental advantages.

Since EVs are propelled by an electric motor (or motors) that is powered by rechargeable battery packs, EVs need to

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be charged periodically. Then the EV charging becomes an important topic [4], [5]. In particular, there are some works on the scheduling of EV charging in literature [6]-[20].

In [6], the EV battery charging behavior was optimized with the objective to minimize charging costs, achieve satisfactory state-of-energy levels, and optimal power balancing. In [7], the problem of optimizing plug-in hybrid electric vehicle (PHEV) charge trajectory (i.e. timing and rate of the charging) was studied to reduce the energy cost and battery degradation. For the purpose of improving the satisfiability of EVs, a reservation-based scheduling algorithm for the charging station to decide the service order of multiple requests was proposed in [8]. In [9], a joint optimal power flow (OPF)-charging (dynamic) optimization problem was formulated with the goal of minimizing the generation and charging costs while satisfying the network, physical and inelastic-load constraints. In [10], utilizing the particle swarm optimization, a proposed algorithm optimally manages a large number of PHEVs charging at a municipal parking station. In [11], the minimization of the waiting time for EV charging via scheduling charging activities spatially and temporally in a large-scale road network was investigated. By modeling an EV charging system as a cyber-physical system, a decentralised online EV charging scheduling scheme was developed in [12]. In [13], the authors formulated the EV charging scheduling problem to fill the electric load valley as an optimal control problem, and a decentralized algorithm was derived. In [14], a strategy to coordinate the charging of plug-in EVs (PEVs) was proposed by using the non-cooperative games [15]. Flexible charging optimization for EVs considering distribution grid constraints, both voltage and power, was investigated in [16]. In [17], the trade off between distribution system load with quality of charging service was considered, and the centralized algorithms to schedule the charging of vehicles were designed. In [18] and [19], real-time scheduling policies of EV charging were considered when both the renewable energy and energy from the grid are available. In [20], the PEV charging and wind power scheduling were integrated, and the synergistic control algorithm of plug-on vehicle charging and wind power scheduling was proposed.

In the paper, we focus on the scheduling approach of EV charging at a charging station. The charging station has multiple charge points and is equipped with renewable energy generation devices and storage battery. The charged energy at a charge point during a period is constant and is called an energy block. We model the arrival of the renewable energy as a Markov chain. The charging energy can also be purchased from power grid, and the price changes also according to another Markov chain. The arrival of the EVs is assumed as a Markov process. Once an EV arrives at the charging station, it

waits in a queue before charging. In each period, the charging station chooses some EVs from the head of the queue for charging. Meanwhile, the station also determines how much energy is supplied from the storage battery (the rest of the required energy is supplied from the power grid). The objective is minimizing the mean waiting time of EVs under the long term cost constraint.

Since the amount of charging energy (i.e., the number of energy blocks to charge) for each EV is random, the scheduling problem is very challenging. We propose queue mapping method to solve the difficulty. We map the EV queue to a charge demand queue. In the charge demand queue, each demand means an energy block that need to charge and some consecutive demands correspond to an EV's required charing energy. We prove that the minimization of the average EV queue length is equivalent to the minimization of the average charge demand queue length. Then we focus on the charge demand queue minimization under the cost constraint. The scheduling problem can be equivalently reconstructed as follows. The demand arrives according to a discrete-time batch Markovian arrival process (D-BMAP) and waits in the charge demand queue before service (charging). In each period, the charging station chooses some demands from the head of the charge demand queue for charging. Meanwhile, the station also determines how much energy is supplied from the storage battery (the rest of the required energy is supplied from the power grid). The objective is minimizing the mean length of the charge demand queue under the long term cost constraint.

Next, we find that the reconstructed optimization problem can be studied under a Markov decision process (MDP) framework. The system state contains the charge demand queue length, the demand arrival, the energy level in the storage battery of the renewable energy, the renewable energy arrival, and the grid power price. Meanwhile, the number of charging demands and the allocated energy from the storage battery constitute the two-dimensional policy. We find that the general case of the reconstructed optimization problem can be analyzed similarly as the analysis of a special case. Then we focus on the analysis of the special case that is formulated as a constrained MDP [21]. We analyze the optimal two-dimensional policy of the constrained MDP by transforming to an average cost MDP and its corresponding discount cost MDP thereafter. First, the constrained MDP is converted to an unconstrained MDP by using Lagrangian relaxation. Moreover, we derive that the optimal solution of the unconstrained MDP with a certain Lagrangian multiplier is the optimal for the original constrained MDP. Next, the unconstrained MDP can be analyzed by transforming to its corresponding discount cost MDP. We obtain two necessary conditions for the optimal solution. Third, we analyze the relations between the two elements of the two-dimensional policy, and find that the number of charging demands[1] is dominant. Thus, we propose a conjecture that the constrained MDP problem can be reduced to a MDP problem with the policy to be the number of charging demands only. We then derive the conditions of the system state when the policy that
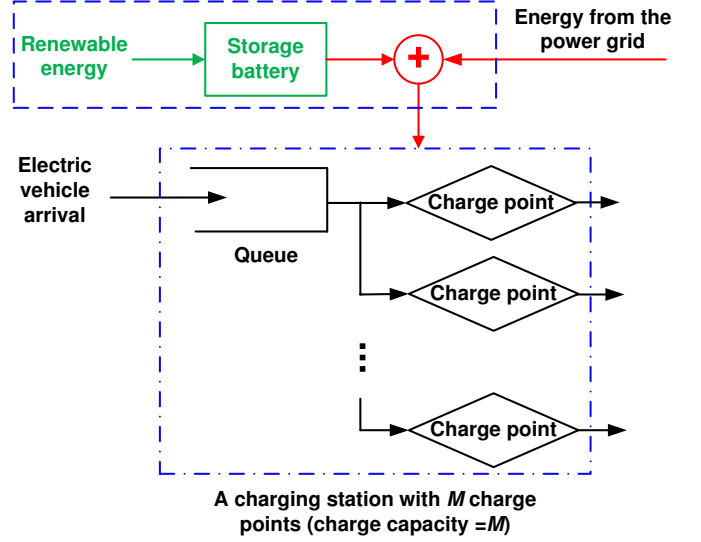

Fig. 1.   System model

charging no demand is optimal. We also obtain the system state conditions when charging as many demands as possible is optimal.

The rest of the paper is structured as follows. In Section II, the system model is described and we formulate an optimization problem that can be studied under the framework of MDP. Section III presents a spacial case of the formulated optimization problem as a constrained MDP to demonstrate the solving process of the general case. Next, we analyze the optimal policy of the constrained MDP in Section IV. In Section V, the numerical results are performed. Finally, Section VI concludes the paper.

II. SYSTEM MODEL AND PROBLEM FORMULATION

Time is divided into periods of length τ each. The EVs arrive at the charging station according to a finite-state ergodic Markov chain $\{A[n]\}$. The EVs wait in a queue before charging as illustrated in Fig. 1. The charging station has M charge points, i.e., at most M EVs can be charged in each period. The charging station has renewable energy generation devices, and it can also gets power from the power grid. The renewable energy is modeled as another finite-state ergodic Markov process $\{E_a[n]\}$. The renewable energy is viewed as free, and the price for the grid power during the n-th period is denoted as $P[n]$. The grid power price remains static during each period and changes between different periods. The sequence of the price, $\{P[n]\}$, is a finite-state ergodic Markov chain. We assume that the charged energy at one charging point during a period is constant, and is denoted as $\mathcal{E}$.[2] In the n-th period, $k[n]$ EVs from the head of the EV queue are allowed to charge. During the n-th period, the charging station allocates $w[n]$ power from the storage battery, and the rest power will be supplied by the power grid. Assume that the required charging energy of the EV, E_c, is independent on

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[1] In the special case, we can use "EV" and "demand" interchangeably.

[2] It is assumed that if an EV utilizes m charge points during a period, the amount of charged energy is $m\mathcal{E}$.
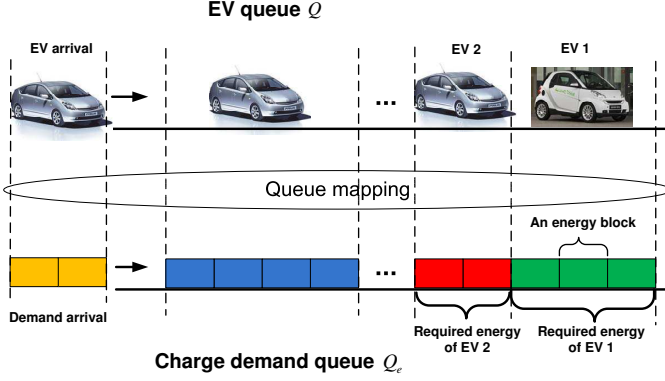
Fig. 2. Queue mapping

each other, and $E_c = L\mathcal{E}$ with L being uniformly distributed in $[1, 2, ..., C]$,[3] i.e., $L \sim \mathcal{U}[1, ..., C]$.

Direct analysis of the EV queue length under the long term cost constraint is difficult due to the randomness of L. We propose the queue mapping method as shown in Fig. 2. Each EV in the EV queue corresponds to several consecutive charge demands (the number of the demands denotes the amount of required energy) in the charge demand queue.[4] The number of EVs at the beginning of the n-th period is $Q[n]$ and the length of the charge demand queue is denoted as $Q_e[n]$. We convert the average EV queue length minimization to the average charge demand queue minimization. Furthermore, we will prove that they are equivalent.

The demand arrival can be given by $A'[n] = \sum_{i=1}^{A[n]} L_i$, where $L_i \sim \mathcal{U}[1, ..., C]$.

Remark: As $\{A[n]\}$ is a Markov chain, we can derive that $\{A'[n]\}$ is a D-BMAP.

In the n-th period, $k'[n]$ demands from the head of the charge demand queue are allowed to charge. During the n-th period, the charging station allocates $w[n]$ power from the storage battery, and the rest power will be supplied by the power grid. Denote the number of charged demands in the n-th period as $K'[n]$. The evolution of the charge demand queue length, $Q_e[n]$, is $Q_e[n+1] = Q_e[n] - K'[n] + A'[n]$. Denote the capacity of the renewable energy storage battery as E_{max}. The stored battery energy at the beginning of the n-th period is $E_b[n]$. The battery energy evolution can be expressed as

$$E_b[n + 1] = \min \{ E_b[n] - W[n]\tau + E_a[n], E_{max} \}$$
$$:= \big(E_b[n] - W[n]\tau + E_a[n] \big)^-. \quad (1)$$

The cost in the n-th period is $\mathcal{C}'[n] = \left(\frac{K'[n]\mathcal{E}}{\tau} - W[n] \right)^+ P[n]$.

Denote the state space as $\mathcal{X}'$ and denote the action space as $\mathcal{A}'$. Let the (random) system state and action in the n-th period be $X'[n] = (Q_e[n], A'[n], E_b[n], E_a[n], P[n]) \in \mathcal{X}'$ and $(K'[n], W[n]) \in \mathcal{A}'$, respectively. Define a policy $\pi' = $

[3]C is a given constant.
[4]A demand means $\mathcal{E}$ energy (i.e., an energy block) need to be charged. In Fig. 2, the first EV (EV 1) in the EV queue wants to charge $3 \times \mathcal{E}$, then it corresponds to the first three consecutive charge demands in the charge demand queue. The second EV (EV 2) charges $2 \times \mathcal{E}$, then it corresponds to the two consecutive charge demands after the first EV's corresponding charge demands.

$(\pi'_0, \pi'_1, \cdots)$ with π'_n generating an action $(k'[n], w[n])$ with a probability [21], [24] at the n-th period. We denote the set of all policies as Π'. Let $x'[n] = (q_e[n], a'[n], e_b[n], e_a[n], p[n])$ be a (fixed) system state. The feasible $(k'[n], w[n])$ in state $x'[n]$ belongs to $\mathcal{K}'(x'[n]) = \{0, 1, \cdots, \min\{q_e[n], M\}\} \times \mathcal{W}(x'[n]) = \{0, \frac{1}{\tau}, \cdots, \frac{e_b[n]}{\tau}\}$.[5] The optimization problem that minimizes the mean charge demand queue length under the long term cost constraint, $\bar{B}$, can be expressed as

$$\min_{\pi \in \Pi'} D_x^{\pi'} := \limsup_{n \to \infty} \frac{1}{n} \mathbb{E}_{x'}^{\pi'} \left[\sum_{i=0}^{n-1} Q_e[i] \right] \quad (2)$$

$$\text{s.t.} \begin{cases} B_x^{\pi'} := \limsup_{n \to \infty} \frac{1}{n} \mathbb{E}_{x'}^{\pi'} \left[\sum_{i=0}^{n-1} \mathcal{C}'[i] \right] \leq \bar{B}, & (3a) \\[2mm] K'[i] \leq \min\{Q_e[i], M\}, & (3b) \\[2mm] W[i] \leq \frac{E_b[i]}{\tau}, & (3c) \end{cases}$$

with initial state $x' = (q_e, a', e_b, e_a, p)$.

Since D-BMAP can be represented by a two-dimensional discrete-time Markov chain (DTMC) [22], the optimization problem in (2) can be analyzed in the framework of MDP. Moreover, the following lemma proves the equivalence of the mean energy demand queue length minimization and the mean EV queue length minimization.

Lemma 1. The minimization of the mean charge demand queue length is equivalent to the minimization of the mean EV queue length.

Proof: See Appendix A. ∎

III. SIMPLIFIED PROBLEM

For conciseness, we give a special case of problem (2) in the section and investigate this relatively simplified problem in the following of the paper to show the solving process. General cases can be analyzed through similar solving process.

When $C = 1$, we have $L_i = 1$. Then the queue mapping is an identity transform and "EV" and "demand" are interchangeable. Thus, we can directly analyze the EV queue using the MDP framework. We have $K[n] = K'[n]$, $A[n] = A'[n]$ and $Q[n] = Q_e[n]$. The queue length evolution is

$$Q[n + 1] = Q[n] - K[n] + A[n]. \quad (4)$$

The battery energy evolution is the same as (1). The cost at the n-th period is given by

$$\mathcal{C}[n] = \left(\frac{K[n]\mathcal{E}}{\tau} - W[n] \right)^+ P[n], \quad (5)$$

where $(\cdot)^+ := \max\{\cdot, 0\}$. The system state becomes $X[n] = (Q[n], A[n], E_b[n], E_a[n], P[n])$ with state space $\mathcal{X}$ and the action is $(K[n], W[n])$ with action space $\mathcal{A}$. $\{X[n], (K[n], W[n])\}$ is a controlled Markov process. Define a policy $\pi = (\pi_0, \pi_1, \cdots)$ that π_n generates an action

[5]The energy has been discretized.

$(k[n], w[n])$ with a probability at the beginning of the n-th period. We denote the set of all policies as Π. The feasible $(k[n], w[n])$ in state $x[n]$ belongs to $\mathcal{K}(x[n]) = \{0, 1, \cdots, \min\{q[n], M\}\} \times \mathcal{W}(x[n]) = \{0, \frac{1}{\tau}, \cdots, \frac{e_b[n]}{\tau}\}$. A stationary deterministic policy is $\pi = (g, g, \cdots)$, where g is a measurable mapping from $\mathcal{X}$ to $\mathcal{K}(x[n]) \times \mathcal{W}(x[n])$. Our objective is to find a policy that minimizes the mean queue delay under the long run constraint on the cost. The optimization problem (i.e., the constrained MDP) is given by

$$\min_{\pi \in \Pi} D_x^\pi := \limsup_{n \to \infty} \frac{1}{n} \mathbb{E}_x^\pi \left[\sum_{i=0}^{n-1} Q[i] \right] \tag{6}$$

$$\text{s.t.} \begin{cases} B_x^\pi := \limsup_{n \to \infty} \frac{1}{n} \mathbb{E}_x^\pi \left[\sum_{i=0}^{n-1} \mathcal{C}[i] \right] \leq \bar{B}, & \text{(7a)} \\ K[i] \leq \min\{Q[i], M\}, & \text{(7b)} \\ W[i] \leq \frac{E_b[i]}{\tau}, & \text{(7c)} \end{cases}$$

where $x = (q, a, e_b, e_a, p) \in \mathcal{X}$ is the initial system state.

Remark: (6) is the special case of (2) with $C = 1$. $C = 1$ means that EVs charge the same amounts of energy, $\mathcal{E}$ (e.g., an EV production company).

IV. ANALYSIS OF THE OPTIMAL POLICY

In this section, we perform theoretical study on the optimal policy. First, we prove that the constrained MDP can be analyzed through an unconstrained MDP. Then, we focus on the analysis of the unconstrained MDP. We analyze the unconstrained MDP by using its corresponding discount MDP. Next, we consider the dimension reduction of the two-dimensional policy. Finally, we propose two stationary deterministic policies based on the theoretical results.

A. Transformation to the unconstrained MDP and discount MDP

Define $f_\beta(x, k, w) := \beta \left(\frac{k\mathcal{E}}{\tau} - w \right)^+ p + q$. We have the following unconstrained MDP (i.e., UP_β).

$$\min_\pi J_\beta^\pi(x) := \limsup_{n \to \infty} \frac{1}{n} \mathbb{E}_x^\pi \left[\sum_{i=0}^{n-1} f_\beta(X[i], K[i], W[i]) \right]. \tag{8}$$

Remark: UP_β is an average cost MDP. Its optimal solution is referred to as the average cost optimal policy.

The following lemma reveals that the constrained problem has the same solution as UP_β with a certain β.

Lemma 2. There exists $\beta > 0$ for which the optimal solution of the unconstrained MDP in (8) (i.e., UP_β) is also optimal for the constrained MDP in (6).

Proof: See Appendix B. ∎

Next, we define a discount cost MDP with discount factor α corresponding to UP_β for each initial system state $x = (q, a, e_b, e_a, p)$, with value function

$$V_\alpha(x) = \min_\pi \mathbb{E}_x^\pi \left[\sum_{i=0}^\infty \alpha^i f_\beta\left(X[i], K[i], W[i]\right) \right]. \tag{9}$$

The optimal solution for the discounted problem is called a discount optimal policy.

The following lemma reveals the existence of the optimal stationary deterministic policy of UP_β, and furthermore, how to derive the average cost optimal policy.

Lemma 3. There exists a stationary deterministic policy (k, w) that solves UP_β, which can be obtained as a limit of discount optimal policies as $\alpha \to 1$.

Proof: See Appendix C. ∎

Based on the above analysis, we find that the constrained MDP can be analyzed through the defined average cost MDP and its corresponding discount cost MDP thereafter. Hence, we first investigate the solution of the discount cost MDP in the following subsection.

B. The discount optimal policy

For state-action pair $(x = (q, a, e_b, e_a, p), (k, w))$, let $u = q - k$ and $\eta = e_b - w\tau$. Then $(u(x), \eta(x))$ can also define a stationary deterministic policy. Then, the discounted cost optimality equation [23], [24] is given by

$$V_\alpha(q, a, e_b, e_a, p) = \min_{\substack{u \in \{0, 1, \cdots, \min\{q, M\}\} \\ \eta \in \{0, 1, \cdots, e_b\}}}$$
$$\left\{ \beta \left(\frac{(q - u)\mathcal{E}}{\tau} - \frac{e_b - \eta}{\tau} \right)^+ p + q \right.$$
$$\left. + \alpha \mathbb{E}_{a, e_a, p} \left[V_\alpha(u + A, A, (\eta + E_a)^-, E_a, P) \right] \right\}, \tag{10}$$

and the corresponding value iteration algorithm (or successive approximation method) is

$$V_{\alpha, n}(q, a, e_b, e_a, p) = \min_{\substack{u \in \{0, 1, \cdots, \min\{q, M\}\} \\ \eta \in \{0, 1, \cdots, e_b\}}}$$
$$\left\{ \beta \left(\frac{(q - u)\mathcal{E}}{\tau} - \frac{e_b - \eta}{\tau} \right)^+ p + q + \alpha \times \right.$$
$$\left. \mathbb{E}_{a, e_a, p} \left[V_{\alpha, n-1}(u + A, A, (\eta + E_a)^-, E_a, P) \right] \right\} \tag{11}$$

with $V_{\alpha, 0}(q, a, e_b, e_a, p) = 0$.

First, regarding $V_\alpha(q, h, a, e_b, e)$, we have the following properties (Property 1 - Property 3).

Property 1. $V_\alpha(q, h, a, e_b, e)$ is an increasing function of q.

Proof: See Appendix D. ∎

Property 2. $V_\alpha(q, a, e_b, e_a, p)$ is a non-increasing function of e_b.

Proof: See Appendix E. ∎

In practice, the allocated renewable energy will not surpass the required charging energy. Thus, $k\mathcal{E} \geq w\tau$, i.e.,

$$\frac{(q - u)\mathcal{E}}{\tau} - \frac{e_b - \eta}{\tau} \geq 0. \tag{12}$$

Property 3. $V_\alpha(q, a, e_b, e_a, p)$ is convex in (q, e_b).

Proof: See Appendix F. ∎

Next, the following two lemmas reveal two necessary conditions for the optimality, respectively.

Lemma 4. In state $x = (q, a, e_b, e_a, p)$, $(u(x), \eta(x))$ is not the discount optimal solution if $u(x) > q - \min\{q, M\}$ and $\eta(x) + e_a > E_{max}$.

Remark: Lemma 4 reveals the sufficient condition for the non-optimality, and it can also be viewed as the necessary condition for the optimality. That is to say, any optimal solutions should not satisfy the condition.

Lemma 5. Denote the discount optimal policy in state $x = (q, a, e_b, e_a, p)$ as $(u^*(x), \eta^*(x))$. Then, $(u^*(x), \eta^*(x))$ satisfies the following inequality array[6]

$$Z_1(u^*, a, \eta^*, e_a, p) \le \beta \frac{\mathcal{E}}{\tau} p \le Z_1(u^* + 1, a, \eta^*, e_a, p), \quad (13)$$

$$Z_2(u^*, a, \eta^*, e_a, p) \le \beta \frac{-p}{\tau} \le Z_2(u, a, \eta^* + 1, e_a, p), \quad (14)$$

$$Z_3(u^*, a, \eta^*, e_a, p) \le \beta \frac{p}{\tau}(\mathcal{E} - 1)$$
$$\le \quad Z_3(u^* + 1, a, \eta^* + 1, e_a, p), \quad (15)$$

where

$$Z_1(u, a, \eta, e_a, p)$$
$$= \alpha \mathbb{E}_{a, e_a, p}\big[G_1(u + A, A, (\eta + E_a)^-, E_a, P)\big] \quad (16)$$

with

$$G_1(q, a, e_b, e_a, p) =$$
$$V_\alpha(q, a, e_b, e_a, p) - V_\alpha(q - 1, a, e_b, e_a, p), \quad (17)$$

$$Z_2(u, a, \eta, e_a, p)$$
$$= \alpha \mathbb{E}_{a, e_a, p}\big[V_\alpha(u + A, A, (\eta + E_a)^-, E_a, P)$$
$$- \quad V_\alpha(u + A, A, (\eta - 1 + E_a)^-, E_a, P)\big], \quad (18)$$

and

$$Z_3(u, a, \eta, e_a, p) =$$
$$\alpha \mathbb{E}_{a, e_a, p}\big[V_\alpha(u + A, A, (\eta + E_a)^-, E_a, P)$$
$$- \quad V_\alpha(u - 1 + A, A, (\eta - 1 + E_a)^-, E_a, P)\big]. \quad (19)$$

Proof: See Appendix G. ∎

Remark: Lemma 5 gives the necessary condition of the discount optimality, i.e., the optimal policy (or policies) should be the solution(s) of the inequality array. Specially, if the inequality array has a single solution, the corresponding single solution is the optimal policy since the existence of the optimal policy.

C. The average cost optimal policy

First, Lemma 4 still holds for the average cost MDP. Next, based on Lemma 3 and Lemma 5, we have the following lemma.

[6]Using Property 3, we can derive that $Z_1(u, a, \eta, e_a, p) \le Z_1(u + 1, a, \eta, e_a, p)$, $Z_2(u, a, \eta, e_a, p) \le Z_2(u, a, \eta + 1, e_a, p)$, and $Z_3(u, a, \eta, e_a, p) \le Z_3(u + 1, a, \eta + 1, e_a, p)$.

Lemma 6. Given state $x = (q, a, e_b, e_a, p)$, the average cost optimal policy $(u^*(x), \eta^*(x))$ should satisfy the following inequality array

$$\tilde{Z}_1(u^*, a, \eta^*, e_a, p) \le \beta \frac{\mathcal{E}}{\tau} p \le \tilde{Z}_1(u^* + 1, a, \eta^*, e_a, p), \quad (20)$$

$$\tilde{Z}_2(u^*, a, \eta^*, e_a, p) \le \beta \frac{-p}{\tau} \le \tilde{Z}_2(u, a, \eta^* + 1, e_a, p), \quad (21)$$

$$\tilde{Z}_3(u^*, a, \eta^*, e_a, p) \le \beta \frac{p}{\tau}(\mathcal{E} - 1)$$
$$\le \quad \tilde{Z}_3(u^* + 1, a, \eta^* + 1, e_a, p), \quad (22)$$

where

$$\tilde{Z}_1(u, a, \eta, e_a, p) = \lim_{\alpha \to 1} Z_1(u, a, \eta, e_a, p),$$

$$\tilde{Z}_2(u, a, \eta, e_a, p) = \lim_{\alpha \to 1} Z_2(u, a, \eta, e_a, p),$$

and

$$\tilde{Z}_3(u, a, \eta, e_a, p) = \lim_{\alpha \to 1} Z_3(u, a, \eta, e_a, p).$$

D. Reducing the policy's dimension

The number of charging EVs k and the power allocation from the battery w are coupled together, they affect each other. However, if we assume that k has been chosen, then the required total power has been fixed. In this case, we will allocate as much power as possible from the battery to meet the required total power, i.e., the greedy policy for the battery power allocation. This is because the power from the battery is free (please refer to (5)). We can guess that the greedy allocation strategy of battery power is the optimal policy. However, it is difficult to prove. The difficulty lies in the fact that the remaining battery energy will affect the future action and cost (e.g., (10)). On the other hand, once w has been fixed, the power allocation from the power grid can also affect k. In summary, when k is chosen, the optimal w^* is the greedy policy. By contrast, if w is fixed, the optimal k is not fixed, we need to solve the power allocation from the power grid to find the optimal k^*. Thus, we can reduce the policy from (k, w) to k. We have the following conjecture.

Conjecture 1. Let $\pi_k = (k[0], k[1], \cdots)$, and (6) can be converted to

$$\min_{\pi_k} B_x^{\pi_k} := \limsup_{n \to \infty} \frac{1}{n} \mathbb{E}_x^{\pi_k} \left[\sum_{i=0}^{n-1} Q[i] \right] \quad (23)$$

s.t.
$$\begin{cases} B_x^{\pi_k} := \limsup_{n \to \infty} \frac{1}{n} \mathbb{E}_x^{\pi_k} \Big[\sum_{i=0}^{n-1} \Big(\frac{K[i]\mathcal{E}}{\tau} \\ \qquad - \frac{1}{\tau} \min\{K[i]\mathcal{E}, E_b[i]\} \Big)^+ P[i] \Big] \le \bar{B}, \quad (24a) \\ K[i] \le \min\{Q[i], M\}, \quad (24b) \end{cases}$$

where the evolution of energy in the battery becomes

$$E_b[i + 1] = (E_b[i] - \min\{K[i]\mathcal{E}, E_b[i]\} + E_a[i])^-. \quad (25)$$

Remark: The policy can be reduced in dimension $((k, w) \to k)$. If the stated β in Lemma 2 satisfying $\beta \gg 1$, Conjecture

1 can be proved based on (10) in addition with Lemma 3 and Lemma 2.

In the following, we discuss the optimal policy after dimension reduction. For state-action pair $(x = (q, a, e_b, e_a, p), k)$, let $u = q - k$, and $u(x)$ can also define a stationary deterministic policy. We have the following lemmas to reveal the properties of the optimal policy.

Lemma 7. Denote the discount optimal policy in state $x = (q, a, e_b, e_a, p)$ as $u^*(x)$. Then, $u^*(x)$ satisfies

$$Z(u^*) \leq \beta \frac{\mathcal{E}}{\tau} p \leq Z(u^* + 1), \quad (26)$$

where

$$Z(u) = \alpha \mathbb{E}_{a, e_a, p} \Big[V_\alpha(u + A, A, (\eta(u) + E_a)^-, E_a, P)$$
$$- V_\alpha(u - 1 + A, A, (\eta(u - 1) + E_a)^-, E_a, P) \Big]$$
$$+ \beta \frac{\eta(u) - \eta(u - 1)}{\tau} p \quad (27)$$

with

$$\eta(u) := \max\{0, e_b - (q - u)\mathcal{E}\}. \quad (28)$$

Furthermore, the average cost optimal policy u^* satisfies

$$\tilde{Z}(u^*) \leq \beta \frac{\mathcal{E}}{\tau} p \leq \tilde{Z}(u^* + 1) \quad (29)$$

with

$$\tilde{Z}(u) = \lim_{\alpha \to 1} Z(u). \quad (30)$$

Proof: See Appendix H. ∎

Lemma 8. For $x = (q, a, e_b, e_a, p)$ satisfying

$$Z(q - \min\{q, M\}) > \beta \frac{\mathcal{E}}{\tau} p, \quad (31)$$

$u = q - \min\{q, M\}$ is the discount optimal policy. In addition, for (q, a, e_b, e_a, p) satisfying

$$Z(q) < \beta \frac{\mathcal{E}}{\tau} p, \quad (32)$$

$u = q$ is the discount optimal policy.

Proof: See Appendix I. ∎

Remark: $u = q - \min\{q, M\}$, i.e., $k = \min\{q, M\}$ means charging as many EVs as possible. If the number of EVs in the queue is less than the charge point number M, charge all the EVs. Otherwise, charge M EVs from the head of the queue. $u = q$, i.e., $k = 0$ denotes charging no EV.

Based on Lemma 8 and Lemma 3, we have

Lemma 9. For $x = (q, a, e_b, e_a, p)$ satisfying

$$\tilde{Z}(q - \min\{q, M\}) > \beta \frac{\mathcal{E}}{\tau} p, \quad (33)$$

$u = q - \min\{q, M\}$ is the average cost optimal policy. In addition, for (q, a, e_b, e_a, p) satisfying

$$\tilde{Z}(q) < \beta \frac{\mathcal{E}}{\tau} p, \quad (34)$$

$u = q$ is the average cost optimal policy.

E. Two stationary deterministic policies

Based on all above theoretical analysis, we propose the following two specific stationary deterministic policies. For state $x = (q, a, e_b, e_a, p)$, we define the radical policy as $\left(k = \min\{q, M\}, w = \frac{\min\{e_b, k\mathcal{E}\}}{\tau}\right)$. That is to say, we charge as many EVs as possible, and use the greedy policy for the battery energy allocation, i.e., if the required energy is not greater than the battery energy, then all the energy will be supplied from the storage battery and no grid power will be used. Otherwise, all the storage battery energy is allocated, and the rest will be supplied from the power grid.

In the radical policy, the average cost constraint is not considered. Then we propose another policy (i.e., the conservative policy) that guarantees the average cost constraint through satisfying the cost constraints in each period. We call the policy $\left(k = \min\{q, M, \frac{e_b + \frac{\bar{B}}{\bar{p}}\tau}{\mathcal{E}}\}, w = \frac{\min\{e_b, k\mathcal{E}\}}{\tau}\right)$ the conservative policy. That is to say, we first guarantee that the cost of charging in each period is less than the average cost constraint, then charge as many EVs as possible and utilize the greedy policy for the battery energy allocation.

In the whole paper, we assume that the power from power grid and renewable energy generator is sufficient to stabilize the queue length. The stability issue such as the bounds on average generation rate of renewable energy or average EV arrival rate will be studied in future work.

V. NUMERICAL RESULTS

In this section, we perform simulations to demonstrate the relations among the mean EV arrival, mean renewable energy arrival, upper bound of the average cost, average cost, and average EV queue length. Meanwhile, we consider different charge point numbers and capacities of the renewable energy storage battery. In the simulations, the period length is $\tau = 1$, and the size of the "energy block" is $\mathcal{E} = 10$.

Fig. 3 shows the average cost performance with respect to the mean EV arrival, $\bar{A}$. In the simulations, we utilize the radical policy. We consider the i.i.d. cases of A, E_a, and P. A takes 0 and $2\bar{A}$ with equal probability 0.5. E_a takes values $\{0, 50, 100\}$ with probabilities $\{0.1, 0.4, 0.5\}$. P takes values $\{5, 10, 20\}$ with probabilities $\{0.2, 0.3, 0.5\}$. The performance is averaged over 10^5 periods. We set the number of charge points $M = 50$ and $M = 8$ in Fig. 3(a) and Fig. 3(b), respectively. Furthermore, we plot the curves for different storage battery capacities: $E_{max} = 100$, $E_{max} = 300$ and infinite capacity, respectively.

In Fig. 3(a), we can see that when $\bar{A}$ is small, the cost is nearly zero. However, when $\bar{A}$ is large (e.g., $\bar{A} \geq 10$), the cost increases rapidly with increase of $\bar{A}$ according to roughly a linear function. It is because when $\bar{A}$ is small, the required energy is small and the battery can supply the energy. Thus, no grid power will be consumed and the cost is zero. Once $\bar{A}$ is larger than a certain value, the required energy is larger than the battery energy, then the grid power will be utilized. As M is large (compared to the considered $\bar{A}$), i.e., the restriction on the number of charge points will not influence the performance, we have $k = \min\{q, M\} = q$ with a high probability. The grid power consumption will increase

with increase of $\bar{A}$. Moreover, when $\bar{A}$ is large, the grid power becomes the main energy source. Based on (5), we derive that the cost varies with $\bar{A}$ roughly according to a linear relation.

From Fig. 3(b), we can find that the average cost is zero when $\bar{A}$ is small, and with increase of $\bar{A}$, the average cost increases. But once $\bar{A}$ is larger than a certain value, the average cost remains constant. It can be explained as follows: when $\bar{A}$ is small, the required energy can be supplied by the battery with a very high probability and no grid power is needed. Then the average cost is zero. When $\bar{A}$ increases, the required energy increases. Once the battery energy is not enough, the grid power will be consumed to fulfill the gap between the required energy and battery energy. With increase of $\bar{A}$, the grid power consumption increases since the average battery energy is constant. Thus, the average cost increases. However, when $\bar{A}$ is large enough, we get $k = \min\{q, M\} = M$ with a high probability because M is not large in this simulations. Then, the required energy $k \times \mathcal{E} = M \times \mathcal{E}$, i.e., it becomes a constant. That means the grid power consumption is a constant also. Thus, the cost remains static.

Fig. 4 depicts the average cost performance with respect to the mean renewable energy arrival, $\bar{E}_a$. The radical policy is applied in the simulations. A takes values 0 and 10 with equal probability 0.5. E_a take values $\{0, \frac{5}{7}\bar{E}_a, \frac{10}{7}\bar{E}_a\}$ with probabilities $\{0.1, 0.4, 0.5\}$, respectively. P is the same as in Fig. 3 and $M = 50$. $E_{max} = 100$, $E_{max} = 300$ and infinite capacity are also respectively considered in the simulations. From the figure, we can find that the cost decreases with increase of $\bar{E}_a$. But once $\bar{E}_a$ is large enough, the cost almost remains static. First, in the range of small $\bar{E}_a$, when $\bar{E}_a$ increases, more free renewable energy will arrive and be stored in the battery. And then, the cost will decrease. If the battery capacity is large enough, all the arrived renewable energy can be stored in the battery. With the increase of $\bar{E}_a$, the battery energy will increase all the time. Once the battery energy is larger than the required energy for charging, no grid power is needed then, and the cost becomes zero since that time. If the battery capacity is not large (e.g., $E_{max} = 100$ in the figure), the overflow occurs when $\bar{E}_a$ is large. That is to say, the battery energy will remain E_{max} even though we increase $\bar{E}_a$. On the other hand, E_{max} is smaller than the required charge energy, so grid power is still needed. Consequently, the cost is non-zero and remains static.

From Fig. 3 and Fig. 4, we can observe that the larger the battery capacity, the lower the cost. That is because when E_{max} is larger, the probability of overflow will be lower (it is zero for infinite capacity). Then, less free renewable energy is wasted and the cost will be lower. Furthermore, we can derive that if $\bar{A}$ is less a certain value or $\bar{E}_a$ is larger than a certain value, the average cost can be less than a certain value, Then we claim that when $\bar{A}$ is less a certain value or $\bar{E}_a$ is larger than a certain value, the radical policy is also optimal even when considering the constraint.[7]

Fig. 5 illustrates the average EV queue length performance with respect to the upper bounds of the average cost when the
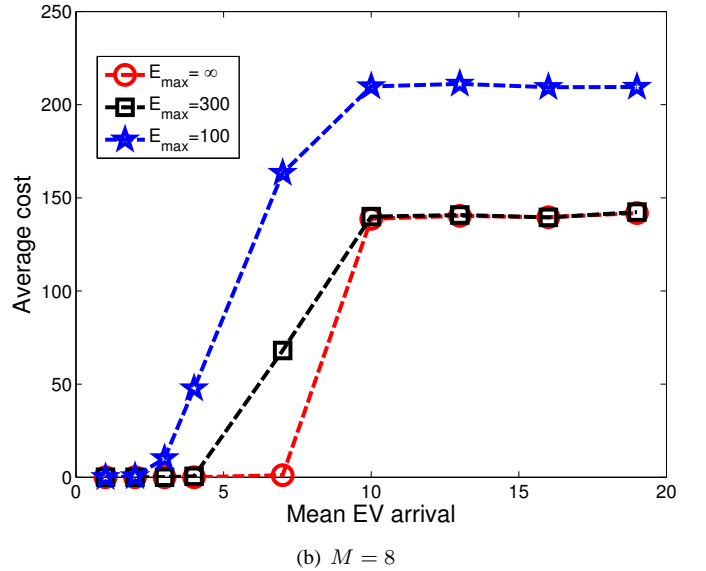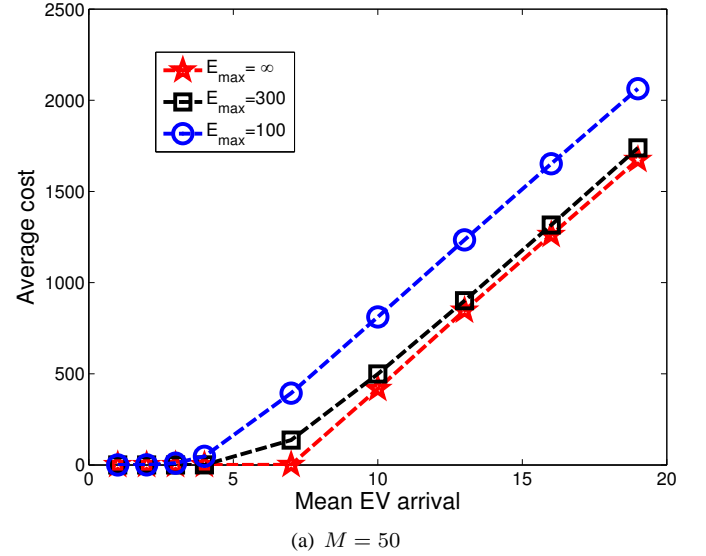
---

[7]Notice that the radical policy is optimal for the mean EV queue delay minimization without the average cost constraint.

(a) $M = 50$

(b) $M = 8$

Fig. 3. The average cost performance v.s. $\bar{A}$ under different values of E_{max}.

conservative policy is applied. In the simulations, A chooses values $\{0, 12\}$ with equal probability 0.5. E_a and P have the same settings as in Fig. 3. In the plotting, we consider different values of the battery capacity and charge point number. We can observe that the average length performance improves with increase of $\bar{B}$, and when $\bar{B}$ is larger than a certain value, the average length performance become almost constant. The reason is as follows: when $\bar{B}$ is small, $k = \min\{q, M, \frac{e_b + \frac{\bar{B}}{p}\tau}{\mathcal{E}}\} = \min\{q, \frac{e_b + \frac{\bar{B}}{p}\tau}{\mathcal{E}}\}$ with a high probability and it increases with increase of $\bar{B}$. Thus, the average EV queue length performance increases. Once $\bar{B}$ is large enough, we get $k = \min\{q, M\}$, and the average length remains static with respect to $\bar{B}$. Additionally, by comparing the four curves, we can derive that the larger the capacity or the charge point number, the better the length performance.
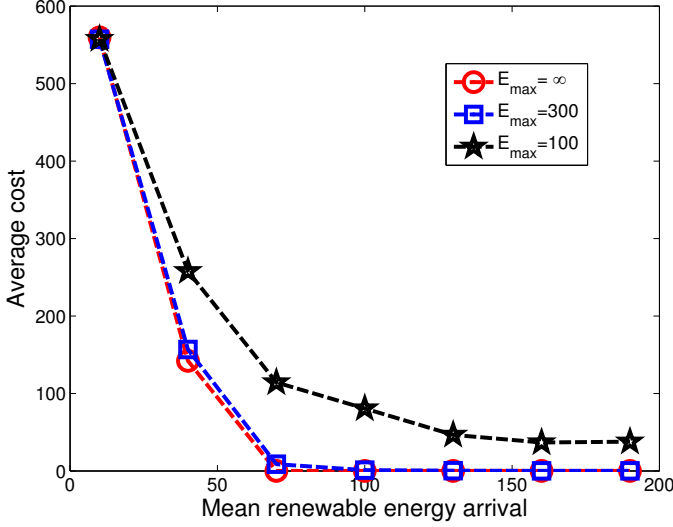
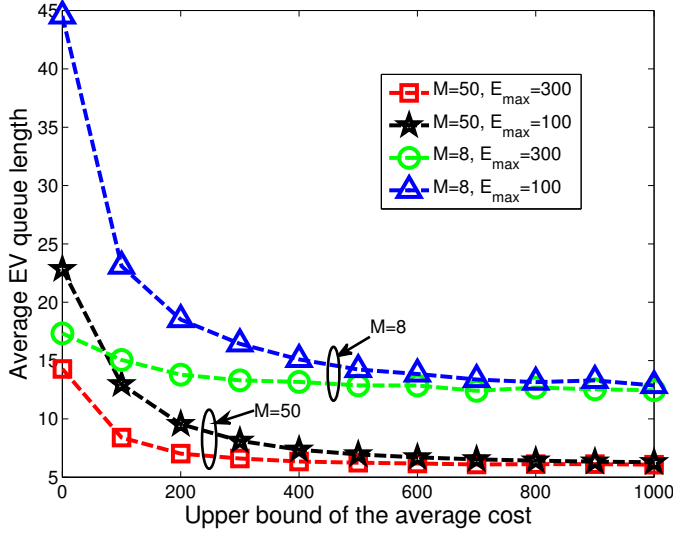Fig. 4.   The average cost under different $\bar{E}_a$ and E_{max}.

Fig. 5.   The average EV length performance v.s. $\bar{B}$.

VI. CONCLUSION

We consider the scheduling of the EVs' charging at a charging station whose energy is provided from both the power grid and local renewable energy. Under the uncertainty of the EV arrival, the renewable energy, the grid power price, and the charging energy of each EV, we study the mean delay optimal scheduling with the average cost constraint. We analyze the optimal policy of the formulated MDP problem. In addition, two specific stationary policies (radical policy and conservative policy) are applied in the simulations to reveal the impacts of relevant parameters on the performance.

APPENDIX A
PROOF OF LEMMA 1

First, the energy demand queue length and the EV queue length have the following relation. $Q_e[n] = \sum_{i=1}^{Q[n]} L_i$ with L_i being irrelevant to the queue state. Thus the average energy demand queue length is $\frac{1}{n}\sum_{j=1}^{n} Q_e[j] = \frac{1}{n}\sum_{j=1}^{n}\sum_{i=1}^{Q[j]} L_i^j$.

Meanwhile, if an EV comes earlier than another EV, it will leave earlier in the EV queue serving. Using the queue mapping mechanism, the earlier arrived EV will leave no later also in the energy demand queue serving.[8] That is to say, the queue mapping is an isotonic mapping. Then, we claim that a policy minimizing the mean EV queue length results in minimal mean demand queue length, and vice versa.

APPENDIX B
PROOF OF LEMMA 2

The proof is based on the results of [25]. We prove that for some β, the optimal policy π^* of the unconstrained MDP (8) (i.e., UP$_\beta$) satisfies 1) π^* yields B^{π^*} and D^{π^*} as limits for all $x \in \mathcal{X}$; 2) $B^{\pi^*} = \bar{B}$. Observe that $\limsup$ and $\liminf$ are equal for each $\beta > 0$ (since the controlled chain is ergodic and the policy is stationary [24]).

APPENDIX C
PROOF OF LEMMA 3

First, we derive that the conditions of Proposition 2.1 in [26] are satisfied. Then a discount optimal stationary policy exists. Next, we prove that for some x_0, $V_\alpha(x) - V_\alpha(x_0) < \infty$. Third, there exits a policy $\pi \in \mathcal{A}$ and an initial state $x \in \mathcal{X}$ such that $J_\beta^\pi < \infty$ in the practical problem. Otherwise, the cost is infinite for all policies and any policy is optimal. Accordingly, we can prove the lemma by applying Theorem 3.8 in [26].

APPENDIX D
PROOF OF PROPERTY 1

We verify the increasing property by induction. According to (11), $V_{\alpha,0} = 0$ and $V_{\alpha,1} = \frac{\beta\left((q-\min\{q,M\})\mathcal{E}-e_b\right)^+ p}{\tau} + q$. The increasing property in q holds. Assume $V_{\alpha,n-1}(q,a,e_b,e_a,p)$ is increasing in q. Depending on the values of M, we have the following two cases.

Case 1: $M \geq q + 1$. Fix (a, e_b, e_a, p), in the state $(q+1, a, e_b, e_a, p)$, the set of feasible u is $\{0, 1, \cdots, q+1\}$ whereas it is $\{0, 1, \cdots, q\}$ for state (q, a, e_b, e_a, p). Consider state $(q+1, a, e_b, e_a, p)$, let the optimal action be (u^*, η^*) with $u^* \in \{0, 1, \cdots, q\}$, hence

$$V_{\alpha,n}(q+1, a, e_b, e_a, p) =$$
$$\beta\Big(\frac{(q+1-u^*)\mathcal{E}}{\tau} - \frac{e_b - \eta^*}{\tau}\Big)^+ p + (q+1) + \alpha \times$$
$$\mathbb{E}_{a,e_a,p}\left[V_{\alpha,n-1}(u^* + A, A, (\eta^* + E_a)^-, E_a, P)\right]. \quad (35)$$

As (u^*, η^*) is feasible in state (q, a, e_b, e_a, p),

$$V_{\alpha,n}(q, a, e_b, e_a, p) \leq \beta\Big(\frac{(q-u^*)\mathcal{E}}{\tau} - \frac{e_b - \eta^*}{\tau}\Big)^+ p + q$$
$$+ \quad \alpha\mathbb{E}_{a,e_a,p}\left[V_{\alpha,n-1}(u^* + A, A, (\eta^* + E_a)^-, E_a, P)\right]$$
$$\leq \quad V_{\alpha,n}(q+1, a, e_b, e_a, p). \quad (36)$$

If (u^*, η^*) with $u^* = q + 1$,

$$V_{\alpha,n}(q+1, a, e_b, e_a, p) = q + 1$$
$$+ \quad \alpha\mathbb{E}_{a,e_a,p}\left[V_{\alpha,n-1}(q + 1 + A, A, (\eta^* + E_a)^-, E_a, P)\right]. \quad (37)$$

---

[8]Leave at the same time is possible.

Meanwhile, since (q, η^*) is feasible in state (q, a, e_b, e_a, p),

$$
V_{\alpha,n}(q, a, e_b, e_a, p) \leq q
$$
$$
+ \alpha \mathbb{E}_{a,e_a,p} \left[V_{\alpha,n-1}(q + A, A, (\eta^* + E_a)^-, E_a, P) \right]
$$
$$
\overset{(a)}{\leq} V_{\alpha,n}(q + 1, a, e_b, e_a, p), \tag{38}
$$

where (a) holds since the induction hypothesis.

Case 2: $M \leq q$. The set of feasible u is $\{0, 1, \cdots, M\}$ in both the state $(q+1, a, e_b, e_a, p)$ and state (q, a, e_b, e_a, p). Then we can prove the increasing property of $V_{\alpha,n}(q, a, e_b, e_a, p)$ by using (35) and (36).

APPENDIX E
PROOF OF PROPERTY 2

Based on (11), the property can be proved through induction. First, we have $V_{\alpha,0} = 0$ and $V_{\alpha,1} = \frac{\beta \left((q - \min\{q, M\}) \mathcal{E} - e_b \right)^+ p}{\tau} + q$. Thus the non-increasing property in e_b holds for $n = 0, 1$. Next, assume $V_{\alpha,n-1}(q, a, e_b, e_a, p)$ is a non-increasing function of e_b. Fix (q, a, e_a, p), for state (q, a, e_b, e_a, p), let (u^*, η^*) be the optimal policy. We get

$$
V_{\alpha,n}(q, a, e_b, e_a, p) = \beta \left(\frac{(q - u^*)\mathcal{E}}{\tau} - \frac{e_b - \eta^*}{\tau} \right)^+ p +
$$
$$
q + \alpha \mathbb{E}_{a,e_a,p} \left[V_{\alpha,n-1}(u^* + A, A, (\eta^* + E_a)^-, E_a, P) \right]. \tag{39}
$$

Since (u^*, η^*) is feasible in state (q, a, e_b+1, e_a, p), we derive

$$
V_{\alpha,n}(q, a, e_b + 1, e_a, p) \leq \beta \left(\frac{(q - u^*)\mathcal{E}}{\tau} - \frac{e_b + 1 - \eta^*}{\tau} \right)^+ p +
$$
$$
q + \alpha \mathbb{E}_{a,e_a,p} \left[V_{\alpha,n-1}(u^* + A, A, (\eta^* + E_a)^-, E_a, P) \right]. \tag{40}
$$

Combing (39) and (40), we get

$$
V_{\alpha,n}(q, a, e_b, e_a, p) \leq V_{\alpha,n}(q, a, e_b + 1, e_a, p).
$$

Then we complete the proof of the property.

APPENDIX F
PROOF OF PROPERTY 3

First, we prove the following proposition.

Proposition 1. For $\phi \in (0, 1)$ and $\forall x_1, x_2, y$, we have $\phi \min\{x_1, y\} + (1 - \phi) \min\{x_2, y\} \leq \min\{\phi x_1 + (1 - \phi)x_2, y\}$.

Proof: The proposition can be verified by considering $\min\{x_1, x_2\} > y$, $\max\{x_1, x_2\} < y$, and $\min\{x_1, x_2\} \leq y \leq \max\{x_1, x_2\}$, respectively. ∎

The convexity is proved by induction. For $n = 0$, $V_{\alpha,0} = 0$ and is convex. Assume $V_{\alpha,n-1}(q, h, a, e_b, e)$ is convex in (q, e_b). Fix (q, a, e_b, e_a, p), let (u_1, η_1) and (u_2, η_2) be the

optimal policy for (q_1, e_{b1}) and (q_2, e_{b2}). Then, we get

$$
\phi V_{\alpha,n}(q_1, a, e_{b1}, e_a, p) + (1 - \phi)V_{\alpha,n}(q_2, a, e_{b2}, e_a, p)
$$
$$
= \phi \left[\beta \left(\frac{(q_1 - u_1)\mathcal{E}}{\tau} - \frac{e_{b1} - \eta_1}{\tau} \right) p + q_1 \right]
$$
$$
+ (1 - \phi)\left[\beta \left(\frac{(q_2 - u_2)\mathcal{E}}{\tau} - \frac{e_{b2} - \eta_2}{\tau} \right) p + q_2 \right]
$$
$$
+ \alpha \mathbb{E}_{a,e_a,p} \left[\phi V_{\alpha,n-1}(u_1 + A, A, (\eta_1 + E_a)^-, E_a, P) \right.
$$
$$
+ (1 - \phi)V_{\alpha,n-1}(u_2 + A, A, (\eta_2 + E_a)^-, E_a, P) \Big]
$$
$$
\overset{(b)}{\geq} \beta \left[\left(\phi(q_1 - u_1) + (1 - \phi)(q_2 - u_2) \right) \mathcal{E} - \left(\phi(e_{b1} - \eta_1) \right. \right.
$$
$$
+ (1 - \phi)(e_{b2} - \eta_2) \Big) \Big] \frac{p}{\tau} + [\phi q_1
$$
$$
+ (1 - \phi)q_2] + \alpha \mathbb{E}_{a,e_a,p} \Big[V_{\alpha,n-1}(\phi u_1 + (1 - \phi)u_2
$$
$$
+ A, A, \phi(\eta_1 + E_a)^- + (1 - \phi)(\eta_2 + E_a)^-, E_a, P) \Big]
$$
$$
\overset{(c)}{\geq} \beta \left[\left(\phi(q_1 - u_1) + (1 - \phi)(q_2 - u_2) \right) \mathcal{E} - \left(\phi(e_{b1} - \eta_1) \right. \right.
$$
$$
+ (1 - \phi)(e_{b2} - \eta_2) \Big) \Big] \frac{p}{\tau}
$$
$$
+ [\phi q_1 + (1 - \phi)q_2] + \alpha \mathbb{E}_{a,e_a,p} \Big[V_{\alpha,n-1}(\phi u_1
$$
$$
+ (1 - \phi)u_2 + A, A, (\phi \eta_1 + (1 - \phi)\eta_2 + E_a)^-, E_a, P) \Big]
$$
$$
\overset{(d)}{\geq} V_{\alpha,n}(\phi q_1 + (1 - \phi)q_2, a, \phi e_{b1} + (1 - \phi)e_{b2}, e_a, p),
$$

where (b) holds because of the convexity of $V_{\alpha,n-1}(q, h, a, e_b, e)$, (c) holds because of Proposition 1 as well as Property 2, and (d) holds since $(\phi u_1 + (1 - \phi)u_2, \phi \eta_1 + (1 - \phi)\eta_2)$ is feasible for $\phi(q_1, a, e_{b1}, e_a, p) + (1 - \phi)(q_2, a, e_{b2}, e_a, p)$.

APPENDIX G
PROOF OF LEMMA 5

Let

$$
S(u, \eta) = \beta \left(\frac{(q - u)\mathcal{E}}{\tau} - \frac{e_b - \eta}{\tau} \right) p + q
$$
$$
+ \alpha \mathbb{E}_{a,e_a,p} \left[V_\alpha(u + A, A, (\eta + E_a)^-, E_a, P) \right]. \tag{41}
$$

First, we have

$$
S(u + 1, \eta) - S(u, \eta) = -\beta \frac{\mathcal{E}}{\tau} p
$$
$$
+ \alpha \mathbb{E}_{a,e_a,p} \big[V_\alpha(u + 1 + A, A, (\eta + E_a)^-, E_a, P)
$$
$$
- V_\alpha(u + A, A, (\eta + E_a)^-, E_a, P) \big] \tag{42}
$$

and

$$
S(u - 1, \eta) - S(u, \eta) = \beta \frac{\mathcal{E}}{\tau} p
$$
$$
+ \alpha \mathbb{E}_{a,e_a,p} \big[V_\alpha(u - 1 + A, A, (\eta + E_a)^-, E_a, P)
$$
$$
- V_\alpha(u + A, A, (\eta + E_a)^-, E_a, P) \big]. \tag{43}
$$

Then applying $S(u^* + 1, \eta^*) - S(u^*, \eta^*) \geq 0$ and $S(u^* - 1, \eta^*) - S(u^*, \eta^*) \geq 0$, we obtain (13). Similarly, as

$$
S(u, \eta + 1) - S(u, \eta) = \beta \frac{p}{\tau}
$$
$$
+ \alpha \mathbb{E}_{a,e_a,p} \big[V_\alpha(u + A, A, (\eta + 1 + E_a)^-, E_a, P)
$$
$$
- V_\alpha(u + A, A, (\eta + E_a)^-, E_a, P) \big] \tag{44}
$$

and

$$S(u, \eta - 1) - S(u, \eta) = \beta \frac{-p}{\tau}$$
$$+ \quad \alpha \mathbb{E}_{a,e_a,p} \big[V_\alpha(u + A, A, (\eta - 1 + E_a)^-, E_a, P)$$
$$- \quad V_\alpha(u + A, A, (\eta + E_a)^-, E_a, P) \big], \tag{45}$$

we can reach (14) from $S(u^*, \eta^* + 1) - S(u^*, \eta^*) \geq 0$ and $S(u^*, \eta^* - 1) - S(u^*, \eta^*) \geq 0$. In addition,

$$S(u + 1, \eta + 1) - S(u, \eta) = \beta \frac{p}{\tau}(1 - \mathcal{E})$$
$$+ \quad \alpha \mathbb{E}_{a,e_a,p} \big[V_\alpha(u + 1 + A, A, (\eta + 1 + E_a)^-, E_a, P)$$
$$- \quad V_\alpha(u + A, A, (\eta + E_a)^-, E_a, P) \big] \tag{46}$$

and

$$S(u - 1, \eta - 1) - S(u, \eta) = \beta \frac{p}{\tau}(\mathcal{E} - 1)$$
$$+ \quad \alpha \mathbb{E}_{a,e_a,p} \big[V_\alpha(u - 1 + A, A, (\eta - 1 + E_a)^-, E_a, P)$$
$$- \quad V_\alpha(u + A, A, (\eta + E_a)^-, E_a, P) \big]. \tag{47}$$

Then, (15) can be obtained by applying $S(u^* - 1, \eta^* - 1) - S(u^*, \eta^*) \geq 0$ and $S(u^* + 1, \eta^* + 1) - S(u^*, \eta^*) \geq 0$.

APPENDIX H
PROOF OF LEMMA 7

First, based on Conjecture 1, we only need to consider the policy set $\{(u, \eta) : (u, \eta = \eta(u)) \cap (u, \eta) \geq (0, 0)\}$. Consequently,

$$S(u, \eta(u)) = \beta \Big(\frac{(q - u)\mathcal{E}}{\tau} - \frac{e_b - \eta(u)}{\tau} \Big) p + q$$
$$+ \quad \alpha \mathbb{E}_{a,e_a,p} \big[V_\alpha(u + A, A, (\eta(u) + E_a)^-, E_a, P) \big]. \tag{48}$$

Then applying $S(u^* + 1, \eta(u^* + 1)) - S(u^*, \eta(u^*)) \geq 0$ and $S(u^* - 1, \eta(u^* - 1)) - S(u^*, \eta(u^*)) \geq 0$, we get $Z(u^*) \leq \beta \frac{\mathcal{E}}{\tau} p \leq Z(u^* + 1)$. Next, using Lemma 3, we reach the second half of the lemma.

APPENDIX I
PROOF OF LEMMA 8

Following the proof of Lemma 7, we can prove the first half of the lemma by contradiction. Specifically, suppose $u = q - \min\{q, M\}$ is not the optimal solution, then $S(u^* - 1, \eta(u^* - 1)) - S(u^*, \eta(u^*)) \geq 0$ should hold. We have $Z\big(q - \min\{q, M\}\big) \leq Z(u^*) \leq \beta \frac{\mathcal{E}}{\tau} p$ and the contradiction occurs. We can verify the second half of the lemma similarly by using contradiction. Assume $u = q$ is not the optimal solution, then $S(u^* + 1, \eta(u^* + 1)) - S(u^*, \eta(u^*)) \geq 0$ should be satisfied. Consequently, we get $Z(q) \geq Z(u^* + 1) \geq \beta \frac{\mathcal{E}}{\tau} p$. The contradiction occurs then.

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